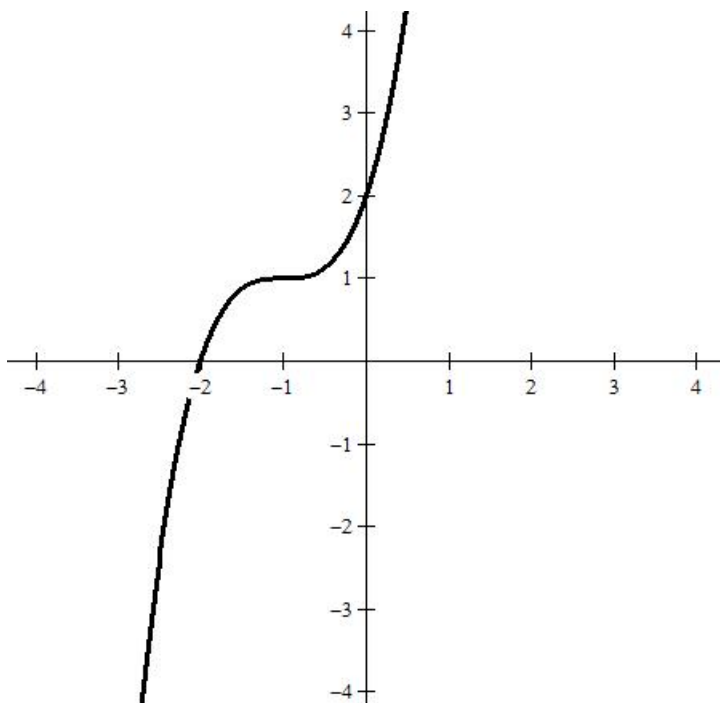


Part A—You may use your calculators on this part of the test.

1. Sketch, as accurately as possible, the inverse of the following function:



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2. Show that the function $f(x) = \frac{3x + 2}{2x - 2}$ has an inverse. (Do NOT find the inverse!)
3. Find the inverse of $f(x) = \frac{3x - 1}{7x + 3}$.
4. Find the inverse of $f(x) = e^{6x-1}$.
5. Find the inverse of $f(x) = \ln(x - 5)$
6. Given $f(x) = \sqrt{x^3 + x^2 + 3x + 4}$, find $(f^{-1})'(3)$.
7. Given that $g(x) = 3 + x + e^x$, find $(g^{-1})'(4)$.
8. Given that $f(x) = 3x^5 + 2x^3$, find $(f^{-1})'(5)$.
9. The rate of growth of the population of a certain city increases at a rate proportional to the population. If the population was 300,000 in 2000 and 350,000 in 2015, find a function which will yield the population at any time t , in years, and use it to predict the population in 2050.

Part B—You may not use your calculators on this part of the test.

10. Given $f(x) = e^{\cos 3x}$, find $f'(x)$.
11. Given $h(x) = \tan^{-1} e^{x^3}$

12. Given $f(x) = \sec^{-1} e^{2x}$, find $f'(x)$.
13. Given $xe^y = y^2 - e^x$, find $\frac{dy}{dx}$.
14. Given $x^3e^y = 3y - 4$, find $\frac{dy}{dx}$.
15. Write an equation of a tangent to $y = x^2 + \ln(2x - 5)$ at $(3, 9)$.
16. Write an equation of a tangent to $y = x + \ln x$ at $(1, 1)$.
17. Write an equation of a tangent to $y = x^2 \ln x$ at (e, e^2) .
18. Given $y = \cos^{-1}(e^{2x})$, find $\frac{dy}{dx}$.
19. Given $f(x) = \sin^{-1} e^{3x}$, find $f'(x)$.
20. Given $f(x) = \tan^{-1}(8x - 3)$, find $f'(x)$.
21. Given $g(x) = \sec^{-1} x^3$, find $g'(x)$.
22. Given $f(x) = 6^{x^3-2x}$, find $f'(x)$.
23. Given $h(x) = x^3 4^{x^2}$, find $h'(x)$.
24. Given $f(x) = \log_7(4x - 1)$, find $f'(x)$.
25. Given $y = \log_3 \sqrt{2x - 1}$, find $\frac{dy}{dx}$.

Answers (not complete solutions)

1. Draw the line $y = x$ and label it. Reflect the given curve across $y = x$.

$$2. f'(x) = -\frac{5}{2(x-1)^2}$$

Since $f'(x) < 0 \forall x \in f$, f is monotonic and therefore has an inverse.

$$3. f^{-1}(x) = \frac{-1-3x}{7x-3} = \frac{3x+1}{3-7x}$$

$$4. f^{-1}(x) = \frac{1+\ln x}{6} \text{ Be careful about argument confusion!}$$

$$5. f^{-1}(x) = e^x + 5$$

$$6. \frac{3}{4}$$

$$7. \frac{1}{2}$$

$$8. \frac{1}{21}$$

$$9. y = 300000e^{(1/15)(\ln(7/6))t}$$

Population in 2050 will be 501507 people

$$10. f'(x) = e^{\cos 3x} (-3 \sin 3x)$$

$$11. h'(x) = \frac{3x^2 e^{x^3}}{1 + e^{2x^3}}$$

$$12. f'(x) = \frac{2e^{2x}}{e^{2x}\sqrt{e^{4x}-1}} = \frac{2}{\sqrt{e^{4x}-1}}$$

$$13. \frac{dy}{dx} = \frac{-e^x - e^y}{xe^y - 2y} = \frac{e^x + e^y}{2y - xe^y}$$

$$14. \frac{dy}{dx} = \frac{-3x^2 e^y}{x^3 e^y - 3} = \frac{3x^2 e^y}{3 - x^3 e^y}$$

$$15. y - 9 = 8(x - 3)$$

$$16. y - 1 = 2(x - 1)$$

$$17. y - e^2 = 3e(x - e)$$

$$18. \frac{dy}{dx} = -\frac{2e^{2x}}{\sqrt{1-e^{4x}}}$$

$$19. f'(x) = \frac{3e^{3x}}{\sqrt{1-e^{6x}}}$$

$$20. f'(x) = \frac{8}{1+(8x-3)^2}$$

$$21. g'(x) = \frac{3x^2}{x^3\sqrt{x^6-1}} = \frac{3}{x\sqrt{x^6-1}}$$

$$22. f'(x) = 6^{x^3-2x} (3x^2 - 2) \ln 6$$

$$23. h'(x) = (x^3) (4^{x^2}) (2x) \ln 4 + 4^{x^2} (3x^2)$$

$$24. f'(x) = \frac{4}{(4x-1) \ln 7} = \frac{4 \log_7 e}{4x-1}$$

$$25. \frac{dy}{dx} = \frac{\frac{2}{2\sqrt{2x-1}}}{\sqrt{2x-1} \ln 3} = \frac{\frac{2}{2\sqrt{2x-1}} \log_3 e}{\sqrt{2x-1}}$$